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## ELECTRO-MAGNETIC THEORY.

*Electro-magnetic Theory.* Vol. ii. By Oliver Heaviside. Pp. xvi + 542. (London: The Electrician Co., Ltd.)

THIS interesting work, the first volume of which appeared some five years ago, well sustains Mr. Heaviside's reputation as an original investigator, and even when we do not agree with his procedure, we must admire his fertility of resource and the skilful manner in which he develops his methods. Although we are more than once warned that the treatment is not formally or logically arranged, as is indeed the case, Mr. Heaviside has nevertheless, in essentials, admirably arranged his matter, so that we are led on by gentle steps from comparatively simple to more complex problems.

The book may be regarded from two distinct points of view. Firstly, without inquiring too closely into the validity of the mathematical methods employed, we may consider the work from a physical point of view as a mathematical theory of the propagation of plane electromagnetic waves in conducting dielectrics, according to Maxwell's theory, or as the theory of the propagation of waves along wires. Secondly, we may consider the book from a purely mathematical point of view as an introduction to the theory of generalised differentiation, divergent series, and Bessel's functions, viewed, however, for the most part through physical spectacles.

The book opens with a discussion of the age of the earth, in which Prof. Perry's results are explained and contrasted with those of Lord Kelvin. Then follows a discussion of the equations

$$-\frac{dV}{dx} = RC, \quad -\frac{dC}{dx} = SpV$$

where  $V$  and  $C$  are the voltage and current,  $R$  and  $S$  the resistance and permittivity per unit of length, and  $p$  stands for  $\frac{d}{dt}$ . A large number of problems are considered in some detail, and it is very noticeable how easily terminal conditions are dealt with by Mr. Heaviside's methods, and in this respect they have a great advantage over Fourier's method. The more general equations

$$-\frac{dV}{dx} = (R + Lp)C, \quad -\frac{dC}{dx} = (K + Sp)V$$

where  $L$  is the inductance and  $K$  the leakance, as Mr. Heaviside terms it, per unit of length, are next considered. These in the case where  $1/R$ ,  $1/L$ ,  $K$  and  $S$  all vary as the  $n$ th power of the distance from  $x=0$  lead to Bessel's functions. As before, a great variety of interesting and important questions are dealt with, and Mr. Heaviside is careful to explain that these are not mere mathematical exercises, but that the formulæ apply to cylindrical electro-magnetic waves. The case of  $R$ ,  $L$ ,  $K$ ,  $S$ , constants is discussed at some length, and owing to the application of the results to practical questions concerning telegraph and telephone cables they should be kept in mind by "practicians." Mr. Heaviside has for long been preaching in the wilderness on this matter, but his labours will bear fruit one day, and we trust that when the day comes it will not be a case of "tulit alter honores," as has happened to other men in other matters.

Some sections are devoted to discussions of the experiments of Dr. Barton and Dr. Bryan, of spherical waves, and, with some reserve, to the experiments of Hertz and Lodge. The sections on spherical waves have, as is pointed out by the author, a practical application in wireless telegraphy. Some rough, but interesting, curves showing the progress of a wave under various circumstances conclude the physical portion of the work.

Passing on to the mathematical aspect of the book, operational methods are freely employed, and their reduction to algebraical form leads us at an early stage of the work to the question of fractional differentiation. This is a subject which has frequently occupied the attention of mathematicians, and two main modes of proceeding have been proposed, one taking  $e^x$ , the other  $x^n$  as the fundamental symbol; the first method was employed by Liouville and Kelland, the second by Peacock. Both methods find formulæ which are certainly true when the index of the operating symbol is an integer, and for the case of the index or fraction both appeal to the principle of the permanence of algebraical forms. If both methods produced the same result in every case all might be well, but most unfortunately this is not so, at least without some further assumption, and it is a question beset with difficulties which system, if either, is to be considered the true one. Mr. Heaviside's method evades rather than elucidates the difficulties. He requires to find the value of  $p^{1/2}$ , where  $p = \frac{d}{dt}$ , and  $t$  is that

function of  $t$  which is zero before and unity after  $t=0$ . To effect this he takes a suitable physical problem, and, solving symbolically, obtains a solution involving  $p^{1/2}$ ; then by another method he finds a solution free from operators; a comparison of the two gives  $p^{1/2} = (\pi t)^{-1/2}$ . This is the same value as is given by Peacock's method, but not that which is given by Liouville's and Kelland's without further assumption. In Chapter vii. another way, on the same lines as before, is given of finding this result, and the remark is added, "I do not give any formal proof that all ways properly followed must necessarily lead to the same result." It is much to be regretted that no hint is given on this point, for, granting that there is a theory of fractional differentiation, the way to be properly followed is the essence of the whole matter.

Some of Mr. Heaviside's methods of dealing with series in Chapter viii. are also open to some objection; he more than once tests the equivalence of two series by giving the variable numerical values and seeing if the two series give the same result. This may be an "excursion to the borders of the realms of duplicity," but scarcely to those of "fearful rigour." It would seem, indeed, from many passages in the book, that Mr. Heaviside considers rigour in mathematics to be of somewhat minor importance; for instance:

"You have first to find out what there is to find out. How you do it is quite a secondary consideration."

If this advice were to be generally followed, mathematicians would no doubt jump many gates in their endeavours to reach the goal on the other side, but whether or no they would not at times land in a quagmire may be open to doubt.

Mr. Heaviside's treatment of Bessel's functions is interesting and suggestive, but the lack of formal arrangement is here severely felt; it is not always easy to distinguish clearly between what is proved and what is experimentally assumed to see how it goes as Mr. Heaviside puts it. The student who is previously unacquainted with the properties of these functions will probably find difficulty in following some of the equations written down without proof. In the equation for  $K_0(gx)$ , p. 226, for example, all the information given about  $\gamma$  (Euler's constant, but not distinguished as such) is "where  $\gamma = 0.5772$  is a certain constant introduced to make  $K_0(gx)$  vanish at infinity"; certain of the conjugate relations are also without proof, but these possibly are left as exercises for the student.

The work is nevertheless one which will well repay careful attention. As has been remarked by Prof. De Morgan :

"The history of algebra shows us that nothing is more unsound than the rejection of any method which naturally arises, on account of one or more apparently valid cases in which such methods lead to erroneous results. Such cases should indeed teach caution, but not rejection."

Mr. Heaviside is much to be congratulated on the light he has thrown on difficult and perplexing questions in both physics and mathematics, and also for calling the attention of mathematicians to a powerful, but somewhat neglected, weapon.

C. S. WHITEHEAD.

#### OUR BOOK SHELF.

*Catalogue of the Lepidoptera of Northumberland, Durham and Newcastle-upon-Tyne.* Part I. By J. E. Robson. *Nat. Hist. Trans. of Northumberland, Durham and Newcastle-upon-Tyne*, Vol. xii. Part 1. Pp. 195.

THE present instalment of this important catalogue includes the butterflies, together with such of the moths as are comprised in the *Sphingina* (hawk-moths), *Bombycina*, and *Noctuina*. In his classification the author thus far follows Mr. Barrett's monograph of the British Lepidoptera, to the unpublished portions of which he has been supplied with references by Mr. Barrett himself. Whatever faults there may be in the scheme of classification in question, and the nomenclature employed therein, the adoption of a uniform system by different writers is highly desirable; and we, therefore, consider that Mr. Robson has been well advised in the course he has adopted.

As the author has had the advantage of the co-operation of all the local collectors of repute, his work may be regarded as a thoroughly up-to-date account of the Lepidopterous fauna of the northernmost counties of England. And how different this fauna is from that of the midland and southern counties may be gathered from a glance at the portion devoted to the butterflies. The common Brimstone Butterfly, for example, is only known in the area treated of by two or three stragglers, its normal range not extending northwards of South Yorkshire. Much more remarkable, however, is the circumstance that certain species of butterflies, such as the Comma and the Red Admiral, which were once common in the two counties, have for the last forty years been extremely scarce, although the second of the two mentioned has once again become a familiar object since 1893. It would be interesting to know the reason why so many of these insects left the district during the

sixties; but on this point the author is silent. On the other hand, as might perhaps have been expected, migratory species, such as the Clouded Yellow and the Camberwell Beauty, which visit England at uncertain intervals in larger or smaller numbers, commonly travel into the northern counties; the author remarking of the last-named insect that it "visits these counties on most of those rare occasions when a wandering horde strikes our shores." Of the moths, it must suffice to say that the Death's-head has occurred in both counties, and there is reason to believe has bred in them, but that the stock is probably maintained by immigration from the south.

The foregoing instances demonstrate that Mr. Robson's work is very far from being a mere dry catalogue; and that it really teems with interesting observations on the life-history and distribution of all the species recorded. If the sequel be maintained at the same high level, the complete catalogue ought to prove a very important contribution to entomological literature.

R. L.

*The Process Year-Book for 1899* ("Penrose's Pictorial Annual"). Edited by William Gamble. Pp. viii + 108. (London: Penrose and Co., 1899.)

THIS is the fifth year's issue of this most excellent review of the graphic arts, and the editor, together with all his co-workers, are to be congratulated on the production of such a handsome and interesting volume.

As in former years, most of the articles are written by those who are at work in some line of process work, and as these are by no means few in number, the reader is made acquainted with a great amount of experience which may help him to success in the future. The feature of the book is undoubtedly the beautiful illustrations, which bring home to the reader the high state of excellence that the art of reproduction has reached at the present day. All kinds of subjects, from a stellar cluster down to an orchid, are illustrated, and these serve as types for showing the results obtained by the working of different processes.

The high order of merit attained should not only render the book a valuable aid to the process worker and others interested in the art of reproduction, but should find many other friends who would delight to possess such a charming collection of high-class illustrations.

*Mathematical Tables.* By James P. Wrapson and W. W. Haldane Gee. Pp. 28. (London: Macmillan and Co., Ltd., 1899.)

THIS set of useful tables in a compact form are abstracted from the compilers' larger volume of "Mathematical and Physical Tables." The idea of this present issue is to place before students tables which are suitable for the class and laboratory, and which give sufficient accuracy for such computations.

To sum up the contents, we have four place logarithms and antilogarithms, natural sines, cosines, and tangents, with interpolation to 1'. Logarithmic sines, cosines, and tangents with differences also to 1'. Tables of squares, exponential functions, weights and measures, and finally a table of conversion for the last mentioned.

*Opinions et Curiosites touchant la Mathématique.* By G. Maupin. (Paris: Carré et Naud, 1898.)

THIS is a collection of curious ideas and essays, which the author has encountered in the course of much heterogeneous reading in ancient scientific works, in which there has been found any reference however remote to mathematical thought. Paradoxes and absurdities alone seem to be considered worth inclusion; the book is of little or no use as a contribution to the history of mathematics.